Fifth Semester B.E. Degree Examination, June/July 2016
Information Theory & Coding

Time: 3 hrs.   Max. Marks:100

Note: *Answer FIVE full questions, selecting*
at least TWO questions from each part.

PART – A

1   a.   Define self information, entropy of the long independent messages, information rate, symbol rate and mutual information.   **(05 Marks)**

    b.   The output of an information source consists of 128 symbols, 16 of which occur with a probability of $\frac{1}{32}$ and the remaining occur with a probability of $\frac{1}{224}$. The source emits 1000 symbols per second. Assuming that the symbols are chosen independently, find the average information rate of this source.   **(05 Marks)**

    c.   For the Markov source model shown in Fig. Q1 (c):
       i)   Compute the state probabilities.
       ii)   Compute the entropy of each state.
       iii)   Compute the entropy of the source.   **(10 Marks)**
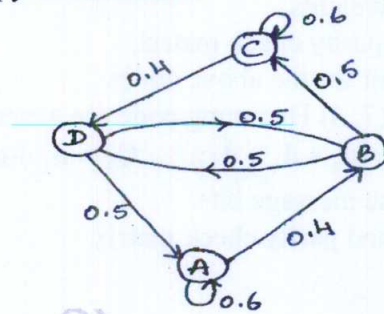

Fig. Q1 (c)

2   a.   State the properties of entropy.   **(04 Marks)**

    b.   A source emits one of the 5 symbols A, B, C, D & E with probabilities $\frac{1}{4}, \frac{1}{8}, \frac{1}{8}, \frac{3}{16}$ and $\frac{5}{16}$ respectively in an independent sequence of symbols. Using Shannon's binary encoding algorithm, find all the code words for the each symbol. Also find coding efficiency and redundancy.   **(08 Marks)**

    c.   Construct a Shannon-Fano ternary code for the following ensemble and find code efficiency and redundancy. Also draw the corresponding code – tree.
$$S = \{S_1, S_2, S_3, S_4, S_5, S_6, S_7\}; \ P = \{0.3, 0.3, 0.12, 0.12, 0.06, 0.06, 0.04\} \ \text{with} \ X = \{0, 1, 2\}$$ **(08 Marks)**

3   a.   Show that $H(X, Y) = H(Y) + H\left(\dfrac{X}{Y}\right)$.   **(05 Marks)**

    b.   The noise characteristics of a non-symmetric binary channel is given in Fig. Q3 (b).   **(10 Marks)**
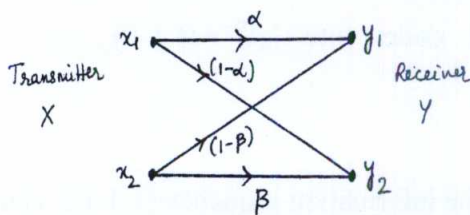

Fig. Q3 (b)

i) Find $H(X)$, $H(Y)$, $H\left(\dfrac{X}{Y}\right)$ and $H\left(\dfrac{Y}{X}\right)$. Given
$$P(x_1) = \frac{1}{4}, \ P(x_2) = \frac{3}{4}, \ \alpha = 0.75, \ \beta = 0.9$$
ii) Also find the capacity of the channel with $r_s = 1000$ symbols/sec.

c. A source has an alphabet consisting of seven symbols A, B, C, D, E, F & G with probabilities of $\frac{1}{4}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{16}$ and $\frac{1}{16}$ respectively. Construct Huffman Quarternery code. Find coding efficiency. **(05 Marks)**

4 a. State Shannon-Hartley theorem and explain its implications. **(08 Marks)**

b. A Gaussian channel has a bandwidth of 4 kHz and a two-side noise power spectral density $\frac{\eta}{2}$ of 10^{-14} watts/Hz. The signal power at the receiver has to be maintained at a level less than or equal to $\frac{1}{10}^{th}$ of milliwatt. Calculate the capacity of this channel. **(06 Marks)**

c. Explain the properties of mutual information. **(06 Marks)**

PART – B

5 a. What are the types of errors and types of codes in error control coding? **(04 Marks)**

b. Consider a (6, 3) linear code whose generator matrix is, $G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$

   i) Find all code vectors.
   ii) Find all the Hamming weights.
   iii) Find minimum weight parity check matrix.
   iv) Draw the encoder circuit for the above codes. **(10 Marks)**

c. The parity check bits of a (7, 4) Hamming code are generated by,
$$C_5 = d_1 + d_3 + d_4; \quad C_6 = d_1 + d_2 + d_3; \quad C_7 = d_2 + d_3 + d_4$$
where d_1, d_2, d_3 & d_4 are the message bits.
   i) Find generator matrix and parity check matrix.
   ii) Prove that $GH^T = 0$. **(06 Marks)**

6 a. Define Binary cyclic codes. Explain the properties of cyclic codes.

b. A (15, 5) linear cyclic code has a generator polynomial,
$$g(x) = 1 + x + x^2 + x^4 + x^5 + x^8 + x^{10}$$ **(08 Marks)**
   i) Draw the block diagram of an encoder for this code $g(x) = 1 + x + x^2 + x^4 + x^5 + x^8 + x^{10}$
   ii) Find the code vector for the message polynomial $D(x) = 1 + x^2 + x^4$ in systematic form.
   iii) Is $V(x) = 1 + x^4 + x^6 + x^8 + x^{14}$ a code polynomial? **(12 Marks)**

7 Write short notes on:
a. BCH codes.
b. RS codes.
c. Golay codes.
d. Brust error correcting codes. **(20 Marks)**

8 a. What are convolutional codes? Explain encoding of convolutional codes using transform domain approach. **(08 Marks)**

b. Consider the (3, 1, 2) convolutional code with $g^{(1)} = (1\ 1\ 0)$, $g^{(2)} = (1\ 0\ 1)$ and $g^{(3)} = (1\ 1\ 1)$
   i) Draw the encoder block diagram.
   ii) Find the generator matrix.
   iii) Find the code word corresponding to the information sequence (1 1 1 0 1) using time domain approach. **(12 Marks)**

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